

Topic 2: Comparing and Ordering Repeating Decimals

for use after **Accentuate the Negative** Investigation 4

Remember that decimals can be *terminating or repeating*. For example, 0.75 (the decimal equivalent to $\frac{3}{4}$) is a terminating decimal.

If you divide 2 by 11, you will get the repeating decimal 0.181818...

You can show that the digits repeat forever by putting a bar over the digits that repeat.

$$0.181818 \dots = 0.\overline{18}$$

Exercises

Find the decimal equivalent to each fraction. Tell if the decimal is terminating or repeating.

1. $\frac{5}{11}$

2. $\frac{2}{3}$

3. $\frac{3}{8}$

4. $\frac{1}{6}$

5. a. Which decimal is greater, $0.\overline{05}$ or $0.\overline{06}$? Explain.
b. Which decimal is greater, $0.\overline{16}$ or $0.\overline{17}$? Explain.
c. Describe a process for deciding which of two positive repeating decimals is greater.

Copy each pair of numbers in Exercises 6–9. Insert $<$ or $>$ to make a true statement.

6. $0.\overline{01}$ \square $0.\overline{001}$

7. 0.75 \square $0.\overline{75}$

8. 8.333 \square $8.\overline{03}$

9. $0.\overline{85}$ \square $0.\overline{805}$

Order each set of decimals from least to greatest.

10. $0.\overline{05}$, $0.1\overline{05}$, $0.00\overline{5}$

11. 0.75 , $0.\overline{705}$, $0.\overline{75}$

12. 2.33333 , 2.033333 , 2

13. $5.\overline{003}$, 5.003 , $5.\overline{03}$

- 14. a.** Which decimal is greater, $-0.0\overline{5}$ or $-0.0\overline{6}$? Explain.
- b.** Which decimal is greater, $-0.1\overline{6}$ or $-0.1\overline{7}$? Explain.
- c.** Describe a process for deciding which of two negative repeating decimals is greater.

Copy each pair of numbers in Exercises 15–18. Insert $<$ or $>$ to make a true statement.

15. $-0.0\overline{1}$ ■ $-0.00\overline{1}$

16. -0.75 ■ $-0.\overline{75}$

17. -4.555 ■ $-4.0\overline{5}$

18. $-0.000\overline{8}$ ■ $-0.\overline{008}$

Order each set of decimals from least to greatest.

19. $-0.0\overline{6}$, $-0.10\overline{6}$, $-0.00\overline{6}$

20. -0.99 , $-0.90\overline{9}$, $-0.900\overline{}$, -1

Comparing and Ordering Repeating Decimals

PACING 1 day

Mathematical Goals

- Compare and order repeating decimals.

Guided Instruction

Students have had some experiences writing fractions as repeating decimals, so Exercises 1–4 help students to recall that information. The remainder of the exercises focus on describing a procedure for comparing two positive repeating decimals, or two negative repeating decimals.

You will have to be careful that students do not incorrectly generalize a procedure for ordering repeating decimals. When you order terminating decimals, like 0.2458 and 0.267, you look for the first decimal place where the digits are not equal and identify the decimal with the greater digit in that place as the greater value. The same procedure works for repeating decimals, provided that the repeating digits are not all 9's. For example, $0.05 > 0.0\overline{4}$, but $0.05 = 0.04\overline{9}$.

Here is the reason for this:

$$\text{Let } x = 0.4\overline{9} \text{ or } 0.4999\dots$$

$$\text{Then } 10x = 4.\overline{9} \text{ or } 4.9999\dots$$

$$\text{So } 9x = 10x - x$$

$$9x = 4.9999\dots - 0.4999\dots$$

$$9x = 4.5$$

$$x = \frac{4.5}{9} \text{ or } \frac{1}{2}$$

Students at this point in CMP2 are not expected to know that $0.\overline{9} = 1$. The examples in these exercises were chosen to avoid that exception.

When students order any numbers, it is often helpful for them to think about which value is farther to the right on a standard number line. The greater of two values is always the value farther to the right on a standard number line. This is especially helpful when ordering negative numbers. For example, $0.1\overline{6} < 0.1\overline{7}$, but $-0.1\overline{6} > -0.1\overline{7}$.

Before Exercise 1, ask:

- *If two fractions have the same decimal representation, are the fractions equivalent?*
- *How many digits are there in a repeating decimal?*
- *How can you tell by looking at the denominator of a proper fraction whether that fraction has a terminating or repeating decimal representation?*

Before Exercise 5, ask:

- *How do you know which of two positive terminating decimals is greater?*
- *How do you know which of two positive repeating decimals is greater?*

Vocabulary

- terminating decimal
- repeating decimal

Guided Instruction

continued

Before Exercise 14, ask:

- How do you know which of two negative terminating decimals is greater?
- How do you know which of two negative repeating decimals is greater?
- How are the procedures for comparing terminating and repeating decimals the same?
- How are the procedures for comparing terminating and repeating decimals different?

Assignment Guide for Topic 2

Core 1–13

Other 14–20

Note: In Exercises 15–19, you may want to ask students to write out six to eight of the decimal digits to be sure that they are repeating the correct string of digits in each case.

Answers to Topic 2

Exercises

1. $0.\overline{45}$, repeating
2. $0.\overline{6}$, repeating
3. 0.375, terminating
4. $0.1\overline{6}$, repeating
5. a. $0.0\overline{6}$; the hundredths place is greater.
b. $0.1\overline{7}$; the hundredths place is greater.
c. Compare corresponding place values of two positive repeating decimals until the digit in one is greater than the other. The decimal with the greater digit is the greater decimal. **Note:** Mathematically, this has to be qualified by saying “so long as the digits after this value are not all 9’s.”
6. $0.\overline{01} > 0.\overline{001}$
7. $0.75 < 0.\overline{75}$
8. $8.333 > 8.0\overline{3}$
9. $0.8\overline{5} > 0.\overline{805}$
10. $0.0\overline{05}$, $0.0\overline{5}$, $0.1\overline{05}$
11. $0.\overline{705}$, 0.75 , $0.\overline{75}$
12. 2, 2.033333 , 2.33333
13. 5.003 , $5.\overline{003}$, $5.\overline{03}$
14. a. $-0.0\overline{5}$; when you graph them on a number line, $-0.0\overline{5}$ is closer to zero than $-0.0\overline{6}$.
b. $-0.1\overline{6}$; when you graph $-0.1\overline{6}$ on a number line, it is closer to zero than $-0.1\overline{7}$.
c. When both decimals are negative, the decimal with the greater absolute value has the lesser value; that is, it is farther to the left of zero.
15. $-0.\overline{01} < -0.\overline{001}$
16. $-0.75 > -0.\overline{75}$
17. $-4.555 < -4.0\overline{5}$
18. $-0.000\overline{8} > -0.\overline{008}$
19. $-0.10\overline{6}$, $-0.0\overline{6}$, $-0.00\overline{6}$
20. -1 , -0.99 , $-0.\overline{909}$, $-0.\overline{900}$